VI.6

(a) From V.6 with $b = m_2 = 0$:

$$(m_c + m_1)\ddot{x} + m_1 l_1 \ddot{\theta}_1 = F(t)$$

$$\ddot{x} + l\ddot{\theta}_1 + g\theta_1 = 0$$

$$(m_c+m_1)s^2 X(s) + m_1 l_1 s^2 \Theta_1(s) = F(s)$$

$$s^2 X(s) + (l_1 s^2 + g)\Theta_1(s) = 0$$

or

$$\begin{bmatrix} (m_c+m_1)s^2 & m_1 l_1 s^2 \\ s^2 & (l_1 s^2 + g) \end{bmatrix} \begin{bmatrix} X(s) \\ \Theta_1(s) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} F(s)$$

(b)
$$\frac{\Theta_1}{F} = \frac{\det \begin{bmatrix} (m_c+m_1)s^2 & 1 \\ s^2 & 0 \end{bmatrix}}{\det \begin{bmatrix} (m_c+m_1)s^2 & m_1 l_1 s^2 \\ s^2 & (l_1 s^2 + g) \end{bmatrix}}$$

$$= \frac{-s^2}{(m_c+m_1)s^2(l_1 s^2+g) - m_1 l_1 s^4}$$

$$= \frac{-s^2}{m_c l_1 s^4 + \cancel{m_1 l_1 s^4} + (m_c+m_1)g s^2 - \cancel{m_1 l_1 s^4}}$$

$$\boxed{\frac{\Theta_1(s)}{F(s)} = \frac{-1}{m_c l_1 s^2 + (m_c+m_1)g}}$$

From (a), $\boxed{\frac{X(s)}{\Theta_1(s)} = -\frac{l_1 s^2 + g}{s^2}}$

(c)