

VI.5

(a) Find equilibria:

EOM

$$(m_c + m_1 + m_2) \ddot{x} + b \dot{x} + m_1 l_1 \cos \theta_1 \ddot{\theta}_1 + m_2 l_2 \cos \theta_2 \ddot{\theta}_2 - m_1 l_1 \sin \theta_1 \dot{\theta}_1^2 - m_2 l_2 \sin \theta_2 \dot{\theta}_2^2 = F(t) \quad (1)$$

$$\ddot{x} \cos \theta_1 + l_1 \ddot{\theta}_1 + g \sin \theta_1 = 0 \quad (2)$$

$$\ddot{x} \cos \theta_2 + l_2 \ddot{\theta}_2 + g \sin \theta_2 = 0 \quad (3)$$

In equilibrium, system is not moving

$$\rightarrow \ddot{x} = \dot{x} = \ddot{\theta}_1 = \dot{\theta}_1 = \ddot{\theta}_2 = \dot{\theta}_2 = 0$$

Subst. into (1), (2), (3):

$$0 = F$$

$$g \sin \theta_1 = 0 \Rightarrow \theta_1 = 0$$

$$g \sin \theta_2 = 0 \Rightarrow \theta_2 = 0$$

} corresponds to loads hanging vertically

$$\boxed{(F_0, \theta_{10}, \theta_{20}) = (0, 0, 0)}$$

(b) Linearize about  $\theta_{10} = \theta_{20} = 0$

$$\begin{aligned} \cos \theta &\approx \cos \theta_0 + \frac{\partial}{\partial \theta} \cos \theta \tilde{\theta} \\ &\approx 1 - \sin \theta_0 \tilde{\theta} \\ &\approx 1 \end{aligned}$$

$$\begin{aligned} \sin \theta &\approx \sin \theta_0 + \frac{\partial}{\partial \theta} \sin \theta \tilde{\theta} \\ &\approx \sin \theta_0 + \cos \theta_0 \tilde{\theta} \\ &\approx \theta \end{aligned}$$

$$\begin{aligned} \sin \theta \dot{\theta}^2 &\approx \sin \theta_0 \dot{\theta}_0^2 + \frac{\partial}{\partial \theta} (\sin \theta \dot{\theta}^2) \Big|_{\theta_0, \dot{\theta}_0} \tilde{\theta} + \frac{\partial}{\partial \theta} (\sin \theta \dot{\theta}^2) \Big|_{\theta_0, \dot{\theta}_0} \tilde{\theta} \\ &\approx 0 \end{aligned}$$

Sub into (1), (2), (3) :

$$(m_c + m_1 + m_2) \ddot{x} + b\dot{x} + m_1 l_1 \ddot{\theta}_1 + m_2 l_2 \ddot{\theta}_2 = F(t)$$

$$\ddot{x} + l_1 \ddot{\theta}_1 + g\theta_1 = 0$$

$$\ddot{x} + l_2 \ddot{\theta}_2 + g\theta_2 = 0$$

7/10/2021