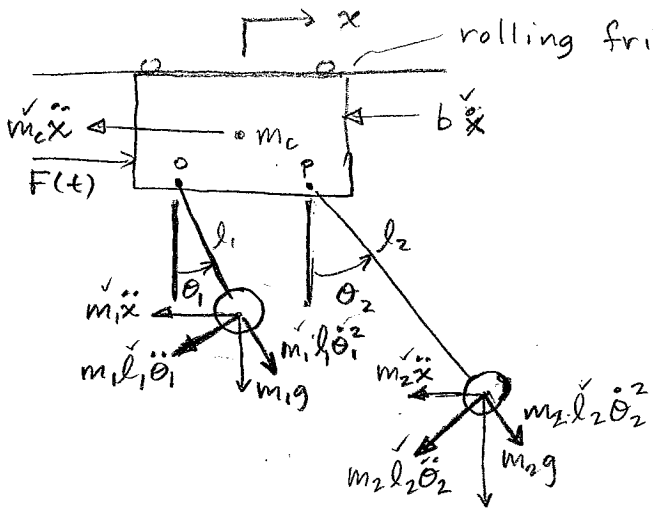
VI.3 Case Study 6 : Double Gantry Crane

(a)



$$\Sigma F_x^* = 0$$

$$(m_c + m_1 + m_2)\ddot{x} + b\dot{x} + m_1 l_1 \cos\theta_1\,\ddot{\theta}_1 + m_2 l_2 \cos\theta_2\,\ddot{\theta}_2 - m_1 l_1 \sin\theta_1\,\dot{\theta}_1^2 - m_2 l_2 \sin\theta_2\,\dot{\theta}_2^2 = F(t)$$

$$\Sigma M_o^* = 0$$

$$m_1\ddot{x}\, l_1 \cos\theta_1 + m_1 l_1^2\ddot{\theta}_1 + m_1 g l_1 \sin\theta_1 = 0$$

$$\Sigma M_p^* = 0$$

$$m_2\ddot{x}\, l_2 \cos\theta_2 + m_2 l_2^2\ddot{\theta}_2 + m_2 g l_2 \sin\theta_2 = 0$$

(b)

$$\begin{bmatrix} (m_c+m_1+m_2) & m_1 l_1\cos\theta_1 & m_2 l_2\cos\theta_2 \\ m_1 l_1\cos\theta_1 & m_1 l_1^2 & 0 \\ m_2 l_2\cos\theta_2 & 0 & m_2 l_2^2 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -b\dot{x} + m_1 l_1 \sin\theta_1\,\dot{\theta}_1^2 + m_2 l_2 \sin\theta_2\,\dot{\theta}_2^2 + F(t) \\ -m_1 g l_1 \sin\theta_1 \\ -m_2 g l_2 \sin\theta_2 \end{bmatrix}$$

(cont.)

$$\begin{bmatrix} \ddot{x} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} (m_c+m_1+m_2) & m_1 l_1 \cos\theta_1 & m_2 l_2 \cos\theta_2 \\ m_1 l_1 \cos\theta_1 & m_1 l_1^2 & 0 \\ m_2 l_2 \cos\theta_2 & 0 & m_2 l_2^2 \end{bmatrix}^{-1} \begin{bmatrix} -b\dot{x} + m_1 l_1 \dot{\theta}_1^2 \sin\theta_1 + m_2 l_2 \dot{\theta}_2^2 \sin\theta_2 + F(t) \\ -m_1 g l_1 \sin\theta_1 \\ -m_2 g l_2 \sin\theta_2 \end{bmatrix}$$

and $\dot{x} = \dot{x}$

$\dot{\theta}_1 = \dot{\theta}_1$

$\dot{\theta}_2 = \dot{\theta}_2$