V. 6 Ball on beam

(a) Linearized EOM

$$\ddot{\tilde{z}} + \frac{5}{7} g \tilde{\theta} = 0$$

$$\left(\tfrac{1}{3} m_2 l^2 + m_1 z_0^2\right) \ddot{\tilde{\Theta}} + m_1 g \tilde{z} = l \tilde{F}$$

Taking laplace x-form:

$$s^2 \tilde{Z}(s) + \frac{5}{7} g \tilde{\Theta}(s) = 0 \qquad (1)$$

$$m_1 g \tilde{Z}(s) + \left(\tfrac{1}{3} m_2 l^2 + m_1 z_0^2\right) s^2 \tilde{\Theta}(s) = l \tilde{F}(s) \qquad (2)$$

$$\underbrace{\begin{bmatrix} s^2 & \frac{5}{7} g \\ m_1 g & \left(\tfrac{1}{3} m_2 l^2 + m_1 z_0^2\right) s^2 \end{bmatrix}}_{A} \begin{bmatrix} \tilde{Z}(s) \\ \tilde{\Theta}(s) \end{bmatrix} = \begin{bmatrix} 0 \\ l \end{bmatrix} \tilde{F}(s)$$

(b)

$$\det A = \underbrace{\left(\tfrac{1}{3} m_2 l^2 + m_1 z_0^2\right)}_{a} s^4 - \frac{5}{7} m_1 g^2 = a s^4 - \frac{5}{7} m_1 g^2$$

$$\frac{\tilde{\Theta}}{\tilde{F}} = \frac{1}{\det A} \cdot \det \begin{bmatrix} s^2 & 0 \\ m_1 g & l \end{bmatrix}$$

$$\boxed{\frac{\tilde{\Theta}}{\tilde{F}} = \frac{l s^2}{a s^4 - \frac{5}{7} m_1 g^2}}$$ where $a = \frac{1}{3} m_2 l^2 + m_1 z_0^2$

Also,

$$\frac{\tilde{Z}}{\tilde{F}} = \frac{1}{\det A} \cdot \det \begin{bmatrix} 0 & 5/7 g \\ l & (\quad) s^2 \end{bmatrix}$$

$$\boxed{\frac{\tilde{Z}}{\tilde{F}} = \frac{-5/7 g l}{a s^4 - \frac{5}{7} m_1 g^2}}$$

$$\boxed{\frac{\tilde{Z}}{\tilde{\Theta}} = \frac{\tilde{Z}/\tilde{F}}{\tilde{\Theta}/\tilde{F}} = \frac{-\frac{5}{7} g}{s^2}}$$

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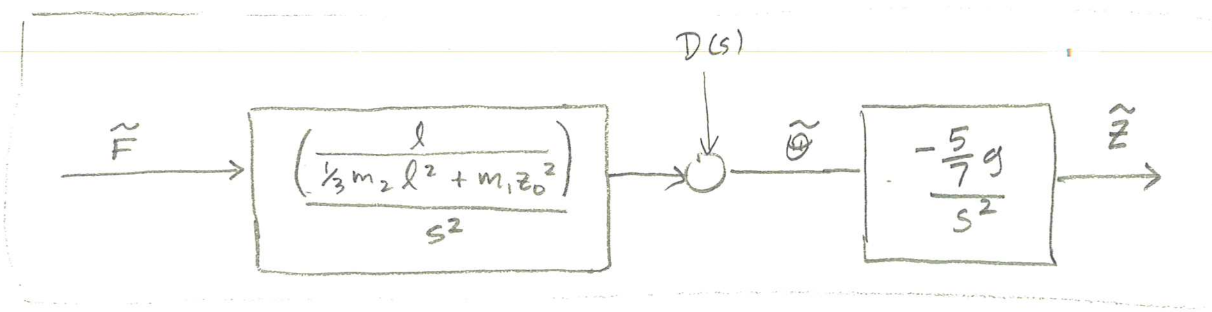
(c) If we neglect the gravity torque of the ball on the beam, then the $m_1 g \tilde{Z}(s)$ term goes away in eqn (2). This gives

$$a s^2 \tilde{\Theta}(s) = \ell \tilde{F}(s)$$

$$\boxed{\frac{\tilde{\Theta}(s)}{\tilde{F}(s)} = \frac{\ell/a}{s^2} = \frac{\left(\frac{\ell}{\frac{1}{3}m_2 \ell^2 + m_1 z_0^2}\right)}{s^2}}$$

(d)



Assumes $\tilde{\Theta}$ and $\tilde{Z}$ are one-way coupled — $\tilde{Z}$ does not affect $\tilde{\Theta}$.