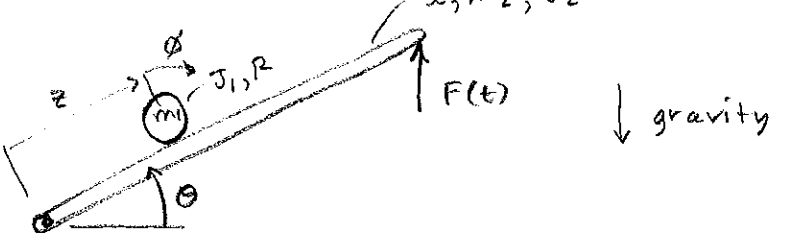
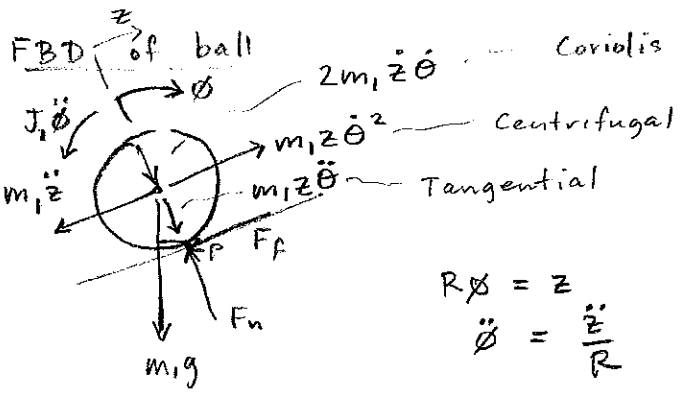
V.3 Case 5 : Ball on Beam

(a)





$$R\phi = z$$

$$\ddot{\phi} = \frac{\ddot{z}}{R}$$

$\Sigma M_P^* = 0$

$$m_1 R(\ddot{z} - z\dot{\theta}^2) + (R\sin\theta)m_1 g + J_1\ddot{\phi} = 0$$

$J_1 = \frac{2}{5}m_1 R^2$

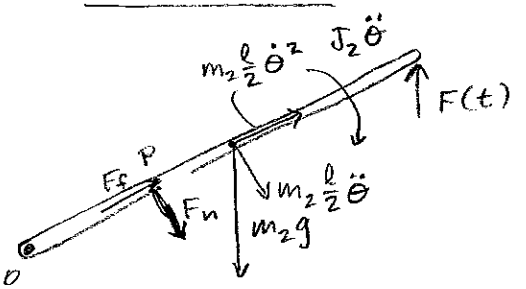
$$\left(\frac{J_1}{R^2} + m_1\right)\ddot{z} - m_1 z\dot{\theta}^2 + m_1 g\sin\theta = 0$$

$$\boxed{\frac{7}{5}\ddot{z} - z\dot{\theta}^2 + g\sin\theta = 0}$$

$\Sigma F_n^* = 0$

$$m_1 z\ddot{\theta} + m_1 g\cos\theta + 2m_1\dot{z}\dot{\theta} = F_n$$

FBD of beam



cont.

$\Sigma M_O^* = 0$

$$J_2\ddot{\theta} + m_2\left(\frac{l}{2}\right)\ddot{\theta} + m_2 g\left(\frac{l}{2}\right)\cos\theta + z\left[m_1 z\ddot{\theta} + m_1 g\cos\theta + 2m_1\dot{z}\dot{\theta}\right] = F(t)\,l\cos\theta$$

$$J_2 = \frac{1}{12} m_2 l^2$$

$$\left(\frac{1}{3} m_2 l^2 + m_1 z^2\right)\ddot{\theta} + 2m_1 z\dot{z}\dot{\theta} + g\cos\theta\left(m_2\frac{l}{2} + m_1 z\right) = F(t)\,l\cos\theta$$

(b) $\dot{z} = \dot{z}$, $\dot{\theta} = \dot{\theta}$ ← first two EOMs

$\dot{z}$: derivative of position; $\dot{z}$: velocity

$$\dot{z} = \dot{z}$$

$$\dot{\theta} = \dot{\theta}$$

$$\ddot{z} = \frac{5}{7} z\dot{\theta}^2 - \frac{5}{7} g\sin\theta$$

$$\ddot{\theta} = \frac{-2m_1 z\dot{z}\dot{\theta} - \left(m_2\frac{l}{2} + m_1 z\right) g\cos\theta + F(t)\,l\cos\theta}{\left(\frac{1}{3} m_2 l^2 + m_1 z^2\right)}$$