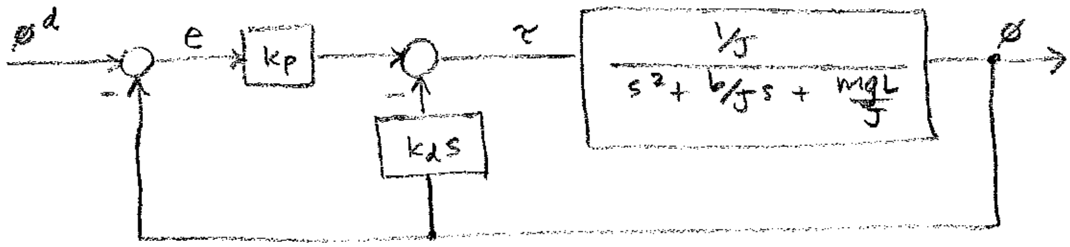
IV. 8

(a)



$$G(s) = \frac{1/J}{s^2 + b/J\, s + \frac{mgL}{J}}$$

$$\tau = k_p(\phi^d - \phi) - k_d s \phi = k_p \phi^d - (k_d s + k_p)\phi$$

$$\phi = G(s)\,\tau$$

$$= \frac{N_G}{D_G}\left[k_p \phi^d - (k_d s + k_p)\phi\right]$$

$$\phi\left[D_G + N_G(k_d s + k_p)\right] = N_G k_p \phi^d$$

$$\phi\left[s^2 + \frac{b + k_d}{J}s + \frac{mgL + k_p}{J}\right] = \frac{k_p}{J}\phi^d$$

$$\boxed{\frac{\phi}{\phi^d} = \frac{k_p/J}{s^2 + \frac{b+k_d}{J}s + \frac{mgL + k_p}{J}}}$$

(b) $\tau_{max} = 50\ \text{N-m}$ $\qquad e_{max} = 0.524\ \text{rad}$

$$k_p \approx \frac{\tau_{max}}{e_{max}} = \frac{50\ \text{N-m}}{0.524\ \text{rad}} = \underline{\underline{95}}\ \text{N-m}$$

(c) CLCE : $s^2 + \frac{b + k_d}{J}s + \frac{mgL + k_p}{J} = 0$

desired CLCE : $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

$$\omega_n = \sqrt{\frac{mgL + k_p}{J}} = \sqrt{\frac{(200\,\text{kg})(9.81\,\text{m/s}^2)(0.03\,\text{m}) + 95}{45\ \text{kg-m}^2}}$$

$$\boxed{\omega_n = 1.85\ \text{rad/s}}$$

(c) cont.

$$\frac{b + k_d}{J} = 2\zeta\omega_n$$

$$k_d = 2J\zeta\omega_n - b$$

$$= 2(45\ \text{kg-m}^2)(0.7)(1.85\ \text{rad/s}) - 5\ \text{N-m-s}$$

$$\boxed{k_d = 112\ \text{N-m-s}}$$

(d) CL poles are roots of

$$s^2 + 2(0.7)(1.849)s + 1.849^2 = 0$$

From Matlab:

$$\boxed{s_{1,2} = -1.296 \pm j\,1.322\ \text{rad/s}}$$