IV.5 Linearize EOM for UUV

(a) Find equilibria:

$$J\ddot{\phi} + b\dot{\phi} + mgL\sin\phi = \tau(t)$$

$$\ddot{\phi} = \dot{\phi} = 0 \rightarrow \boxed{mgL\sin\phi_e = \tau_e}$$

* Specify ϕ_e, then calculate τ_e

- For this system, it would by typical to linearize about the state resulting when $\tau_e = 0$

$$\rightarrow mgL\sin\phi_e = 0$$

$$\rightarrow \underline{\phi_e = 0 \text{ deg}}$$

(b) $f = \sin\phi$

$$\approx \sin\phi_e + \frac{\partial}{\partial\phi}\sin\phi\Big|_{\phi=\phi_e}(\phi - \phi_e)$$

$$\approx \sin\phi_e + (\cos\phi_e)(\phi - \phi_e)$$

$$\approx \phi \quad \text{for } \phi_e = 0$$

Linearized EOM:

$$\boxed{\begin{aligned} \dot{\Omega} &= -\frac{b}{J}\Omega - \frac{mgL}{J}\phi + \frac{1}{J}\tau(t) \\ \dot{\phi} &= \Omega \end{aligned}}$$

$\tilde{\tau} = \tau - \tau_e^{\,0}$

$\tilde{\phi} = \phi - \phi_e^{\,0}$

$\tilde{\Omega} = \Omega - \Omega_e^{\,0}$