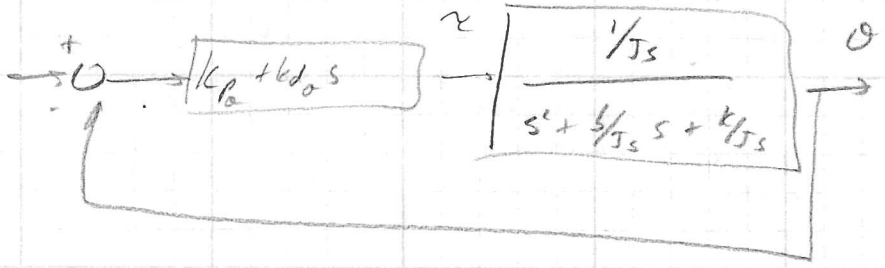
a) Inner Loop



Loop gain is

$$L(s) = (k_{p_a} + k_{d_a} s)\left(\frac{1/J_s}{s^2 + b/J_s s + k/J_s}\right)$$

System is type 0

tracking error to a unit step input is

$$e_{ss} = \lim_{s\to 0} \frac{1}{1 + L(s)} = \frac{1}{1 + \frac{k_p/J_s}{k/J_s}} = \frac{k}{k + k_p}$$

With an integrator, the loop gain is

$$L(s) = \left(\frac{k_{d_a} s^2 + k_{p_a} s + k_{I_a}}{s}\right)\left(\frac{1/J_s}{s^2 + b/J_s s + k/J_s}\right)$$

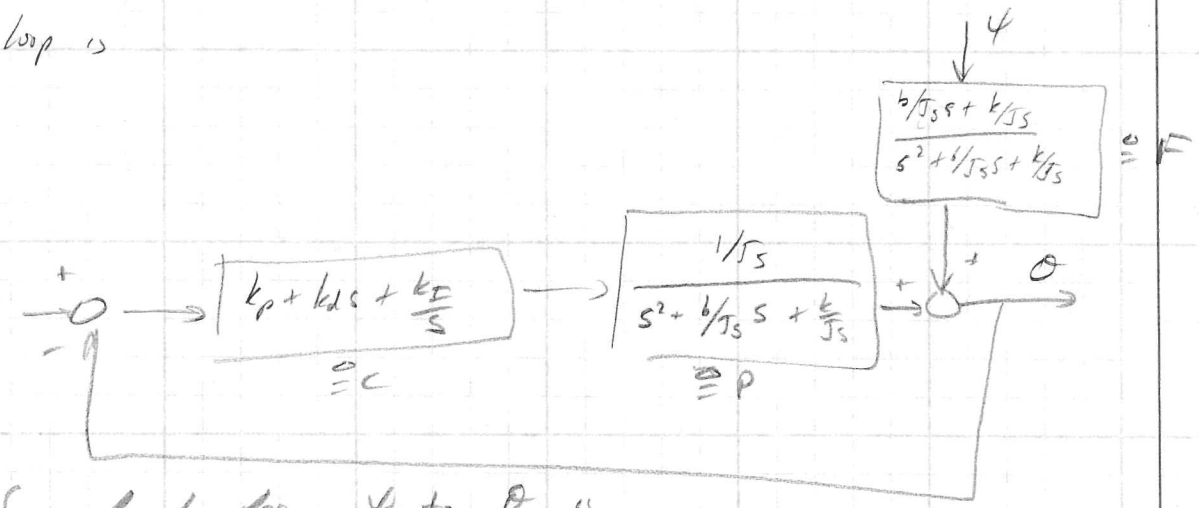
System is type 1

tracking error to a step is zero

tracking error to a ramp is

$$e_{ss} = \lim_{s\to 0} \frac{1}{s L(s)} = \frac{1}{\frac{k_I/\theta/J_s}{k/J_s}} = \frac{k}{k_{I_a}}$$

3) Inner loop is



The transfer function from Ψ to Θ is

$$\Theta = F\Psi - PC\Theta \quad \Rightarrow \quad \Theta(s) = \frac{F}{1+PC}\Psi(s)$$

∴ for a unit step on Ψ(s), the steady state value of Θ is

$$\Theta_{ss} = \lim_{t\to\infty} \Theta(t) = \lim_{s\to 0} s\Theta(s) = \lim_{s\to 0} \frac{s\,F(s)}{1+P(s)C(s)}\frac{1}{s} = \lim_{s\to 0} \frac{F(s)}{1+PC}$$

$$= \lim_{s\to 0} \frac{\frac{b/J_s\, s + k/J_s}{s^2 + b/J_s\, s + k/J_s}}{1 + \left(k_p + k_d s + \frac{k_I}{s}\right)\left(\frac{1/J_s}{s^2 + b/J_s\, s + k/J_s}\right)} = \lim_{s\to 0} \frac{1}{1 + \left(k_p + \frac{k_I}{s}\right)\left(\frac{1}{k}\right)}$$

$$= \lim_{s\to 0} \frac{k}{k + k_p + \frac{k_I}{s}}$$

Without the integrator, the response to a step on Ψ is

$$\Theta_{ss} = \frac{k}{k+k_p}$$

With the integrator, the response is

$$\Theta_{ss} = 0$$

c) The characteristic equation is

$$1 + \left(k_p + k_d s + \frac{k_I}{s}\right)\left(\frac{1/J_s}{s^2 + b/J_s s + k/J_s}\right) = 0$$

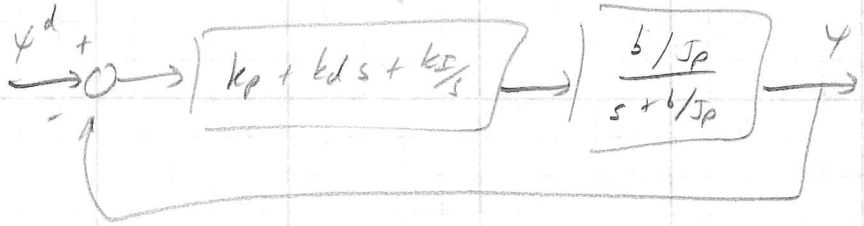
$$\Rightarrow \quad s^2 + \frac{b}{J_s} s + \frac{k}{J_s} + \frac{k_p}{J_s} + \frac{k_d s}{J_s} + \frac{k_I}{J_s}\frac{1}{s} = 0$$

$$\Rightarrow \quad 1 + k_I \left(\frac{1/J_s}{s\left(s^2 + \left(\frac{b+k_d}{J_s}\right)s + \left(\frac{k+k_p}{J_s}\right)\right)}\right) = 0$$

Evans form

rlocus($\left[\frac{1}{J_s}\right]$, $[1, \frac{b+k_d}{J_s}, \frac{k+k_p}{J_s}, 0]$)

d) Outer loop



The loop gain is $L(s) = \left(k_p + k_d s + \frac{k_I}{s}\right)\left(\frac{b/J_p}{s + b/J_p}\right)$

∴ system is type 0 without an integrator and

ss error to step is $\lim_{s\to 0} \frac{1}{1 + L(s)} = \frac{1}{1 + k_p}$

With integrator, the system is type 1 and

ss error to ramp is $\lim_{s\to 0} \frac{1}{sL(s)} = \frac{1}{K_I}$

e) The characteristic equation is

$$1 + \left(k_p + k_d s + \frac{k_I}{s}\right)\left(\frac{b/J_p}{s + b/J_p}\right) = 0$$

$$\Rightarrow \quad s + \frac{b}{J_p} + \frac{k_p b}{J_p} + \frac{k_d b}{J_p} s + \frac{k_I b}{J_p}\frac{1}{s} = 0$$

$$\Rightarrow \quad 1 + K_I \left(\frac{b/J_p}{s\left(\left(1 + \frac{k_d b}{J_p}\right)s + \left(\frac{b + k_p b}{J_p}\right)\right)}\right) = 0$$

Evans form

$$\text{rlocus}\left(\left[\frac{b}{J_p}\right], \left[\left(1 + \frac{k_d b}{J_p}\right), \left(\frac{b + k_p b}{J_p}\right), 0\right]\right)$$