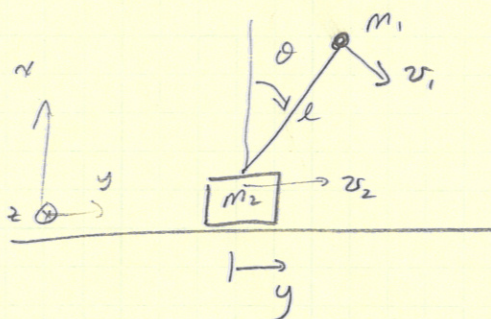


Case Study II

Pendulum on Cart



Assume massless rod
(ie no inertia)

$$K = \frac{1}{2} m_1 \vec{v}_1^T \vec{v}_1 + \frac{1}{2} m_2 \vec{v}_2^T \vec{v}_2$$

where $\vec{v}_2 = \begin{pmatrix} 0 \\ \dot{y} \\ 0 \end{pmatrix}$

and $\vec{v}_1 = \begin{pmatrix} -l\dot{\theta} \sin\theta \\ \dot{y} + l\dot{\theta} \cos\theta \\ 0 \end{pmatrix}$

$$\Rightarrow K = \frac{1}{2} m_1 \left[(-l\dot{\theta} \sin\theta)^2 + (\dot{y} + l\dot{\theta} \cos\theta)^2 \right] + \frac{1}{2} m_2 \dot{y}^2$$

$$= \frac{1}{2} m_1 \left(l^2 \dot{\theta}^2 \sin^2\theta + \dot{y}^2 + 2l\dot{y}\dot{\theta} \cos\theta + l^2 \dot{\theta}^2 \cos^2\theta \right) + \frac{1}{2} m_2 \dot{y}^2$$

$$K = \frac{1}{2} (m_1 + m_2) \dot{y}^2 + \frac{1}{2} m_1 l^2 \dot{\theta}^2 + m_1 l \dot{y} \dot{\theta} \cos\theta$$